Effects of streamwise conduction on thermal performance of nanofluid flow in microchannel heat sinks

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A B S T R A C T
Effects of streamwise conduction on the thermal performance of nanofluid flow in microchannel heat sinks under exponentially decaying wall heat flux are investigated. By employing the first-law principles, models with and without streamwise conduction term in the energy equation are developed for hydrodynamically fully-developed flow. Closed-form solutions are obtained and the analysis emphasizes on the details of discrepancy induced by streamwise conduction between the two models on the heat transport characteristics in nanofluids. The effects of the variations of Peclet number and nanoparticle volume fraction on the thermal characteristics of nanofluid flow in microchannel heat sinks are analyzed and discussed. Due to the tremendous increase in the effective thermal conductivity, the streamwise conduction effect is justified to be more significant in the nanofluid compared to its base fluid. The significance of the streamwise conduction which is prevalent in low-Peclet-number flow is greatly amplified when the volume fraction of nanoparticle is increased. At low Peclet number, the contribution of the streamwise conduction in nanofluid is found to be more than twofold of that in its base fluid. The effect of streamwise conduction on the nanofluid flow in microchannel heat sink is significant albeit not dominant particularly for low Peclet number and high nanoparticle volume fraction of the nanofluid.

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1. Introduction

The unprecedented growth in electronics technologies, especially the miniaturization of the communication and computing devices, poses a challenging problem in the thermal management of such devices. Micro-scale heat transfer becomes a topical subject and the microchannel heat sink has manifested itself as one of the promising solutions to the thermal problem in light of its high thermal performance. To further enhance the thermal performance of the microchannels, the choice of working fluid has been a key issue. The addition of suspended nano-scale solid particles in the base fluid has been reported to be able to enhance the heat transfer characteristic of the conventional fluids. Choi [1] first coined the term “nanofluid” for this type of fluid which exhibits anomalous increase in the effective thermal conductivity even with a small volume fraction of nanoparticle suspension. Reviews of studies on the heat transfer characteristics of nanofluids have been well documented [2–8]. Numerous theoretical studies have been performed to investigate various effects related to the hydrodynamic and thermal characteristics of nanofluids [9–18]. Unlike the micro-sized particles, the use of the ultrafine nanoparticle is free from clogging problem in microchannel flows. Therefore, nanofluid is a promising candidate of innovative working fluid in microchannel flows for the sake of heat transfer enhancement.

Most of the existing analytical studies on microchannel heat sink neglected the effect of streamwise conduction despite the fact that this effect is justified to significantly affect the heat transport rate in proximity to the entrance region of the fluid flow for conventional size channels [19–21]. The streamwise conduction effect appears to be pronounced at the entrance region but diminishes further away from the entrance [22]. For small Peclet number, the characteristic time of convection and conduction is comparable and hence the streamwise conduction becomes indispensable. It was reported that significant error can be generated by neglecting the streamwise conduction for a Peclet number value of 10 [23]. The Nusselt number correlations have been recommended for low-Peclet-number flow by taking into account the effect of the streamwise conduction [24]. Due to the fact that the internal flows in micro-scale devices are typically characterized by finite Peclet numbers, the incorporation of the effect of streamwise conduction is a necessity in the thermal analysis of microchannel flow. The inclusion of streamwise conduction generally involves the Graetz-type problems which are inherently complicated due to the presence of the non-self-adjoint eigenvalues and the non-orthogonal eigenfunctions [25]. To evade the mathematical complexity, the streamwise conduction effect in microchannel flow can be characterized analytically by averaging the fluid temperature over the

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microchannel cross section, leading to a one-dimensional energy equation for the incompressible fluid flow with constant physical properties [26]. The presence of streamwise conduction increases the difference between the wall-fluid temperature and reduces the Nusselt number value within the entrance section. Similar results were obtained by employing the fin approach with a one-dimensional analysis on the streamwise conduction where the fluid temperature distributions exhibit significant discrepancy with those of neglecting the streamwise conduction effect when the Peclet number is sufficiently small for different channel aspect ratio and fluid-to-wall thermal conductivity ratio [27]. The incorporation of streamwise conduction in microchannel flow can also be modeled by employing a porous-medium model which was solved numerically using finite difference method [28]. Comparison was done with the analytical solutions by neglecting the streamwise conduction. Numerical surrogate optimization analysis with an evolutionary algorithm using commercial CFD software has also been performed to study the effect of the streamwise conduction in microchannel heat sinks [29].

Judging from the tremendous increase in the effective thermal conductivity of nanofluids, it is instructive to take the effect of streamwise conduction into account in the evaluation of the heat transfer rate of nanofluid flow in microchannels. In the existing literature, none of the studies deals explicitly with the effect of streamwise conduction on nanofluid flow in microchannels, in spite of the anomalous increase in the effective thermal conductivity of nanofluids which in turn contributes significantly to the streamwise conduction. Such effect is essential in understanding the mechanism of heat transfer processes and predicting the heat transfer rate of nanofluid flow in micro-scale channels. The present study, a basic investigation in filling this gap, emphasizes details of the streamwise conduction heat transfer rates of nanofluid flow in a circular microchannel with exponential wall heat flux thermal boundary condition. Closed-form solutions for the temperature distributions are obtained by solving the governing equations analytically. With the variations of the Peclet number and the nanoparticle volume fraction, the role of the streamwise conduction heat transfer is investigated and its contribution is compared with that of the convection heat transfer. The effect of streamwise conduction on the total heat transfer of nanofluid is scrutinized and compared with that of its base fluid. The underlying physical significance of the streamwise conduction in nanofluid flow in microchannel heat sink is discussed.

2. Mathematical formulation

Nanofluid poses distinct thermal behavior from the conventional fluid associated with its three distinguished main transport properties: thermal conductivity, heat capacity, and viscosity. The addition of the ultra-fine nanoparticles into the base fluid changes these transport properties prominently, enhancing the thermal performance of the nanofluid. The effective thermal conductivity of nanofluid is higher than that of its base fluid with suspension of a small volume fraction of nanoparticles. In the present study, the nanofluid of water-alumina is chosen and its effective thermal conductivity can be predicted using the correlation taking into account the Brownian motion-induced convection from multiple nanoparticles as [30].

$$k_{\text{eff}} = C_k k_f,$$

where $C_k$ is a constant coefficient defined as

$$C_k = \left( 1 + \frac{A R e_{in}^{0.6} P r}{Pr^{0.3}} \phi \right) \frac{k_f (1 + 2 \alpha) + 2 + 2 \phi [k_f (1 - \alpha) - 1]}{k_f (1 + 2 \alpha) + 2 - \phi [k_f (1 - \alpha) - 1]}.$$  

In Eq. (2), $\kappa = k_{\text{eff}} / k_f$ is the thermal conductivity ratio of the thermal conductivity of the particle $k_f$ to the thermal conductivity of the base fluid $k_p$, $Pr = C_{Pr} R h / k_f$ is the Prandtl number, $\phi$ is the nanoparticle volume fraction, $\alpha = 2 R_b / d$ is defined as the Biot number of the particle, with $d$ is the diameter of nanoparticle and $R_b$ is the
interfacial resistance. The Brownian–Reynolds number is given as

\[ R_b = \sqrt{18k_BT/\sigma D_k}, \]

with \( k_b \) the Boltzmann constant and \( T \) the fluid kinetic viscosity. For \( \text{Al}_2\text{O}_3 \) nanoparticles in water as base fluid, it is found that \( R_b = 0.77 \times 10^{-8} \text{ K m}^2 \text{ N}^{-1} \). \( m = 2.5 \) and \( A = 40,000 \) [30]. The nanoparticle absolute velocity which can be viewed as the sum of the base fluid velocity and a nonzero slip velocity is affected by several slip mechanisms namely, Brownian diffusion, inertia, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling [31]. For laminar flows, the relative (slip) velocity between the nanoparticles and the base fluid, the effective viscosity is modeled using a modified Einstein model as [33].

\[ \mu_{\text{eff}} = C_\mu \mu_f, \]

where \( \mu_f \) is the dynamic viscosity of the base fluid and the ratio \( C_\mu \) is defined as

\[ C_\mu = (1 + 2.5\phi) \left[ 1 + \eta \left( \frac{d}{\bar{D}} \right)^{2\phi/3(1+\phi)} \right], \]

with \( \phi = 1/4 \) and \( \eta = 280 \) the empirical constants for \( \text{Al}_2\text{O}_3 \) nanoparticles, and \( D \) the inner diameter of microchannel. In the absence of theoretical formulars to satisfactorily predict the thermal conductivity and the viscosity of nanofluids, the semi-empirical and empirical correlations have been employed. By taking into account the slip velocity between the nanoparticles and the base fluid, the effective viscosity is modeled using a modified Einstein model as [33].

\[ \rho_{\text{eff}} = \rho_f (1 - \phi) + \rho_p \phi, \]

while the effective specific heat, which is more accurate on the basis of mass average, is given by [35].

\[ c_{p,\text{eff}} = \frac{\rho_f c_{p,f}(1 - \phi) + \rho_p c_{p,p} \phi}{\rho_f (1 - \phi) + \rho_p \phi}. \]

The fluid flowing in microchannel is considered as continuous media and the continuum modeling approach is applicable for most of the practical cases with diameter of channel larger than 1 \( \mu \text{m} \) [36]. Based on the viscosity measurements, most of the nanofluids manifest themselves as Newtonian fluids [37,38]. For a steady, hydrodynamically developed laminar and axisymmetric flow in a circular microchannel with an internal radius \( r_0 \), the equation of motion is given by

\[ \frac{\partial}{\partial r} \left( \frac{\partial u}{\partial r} \right) = \frac{1}{\mu_{\text{eff}}} \frac{\partial p}{\partial r}, \]

where the axial pressure gradient \( \partial p/\partial x \) is a constant. The power required to pump the nanofluid is defined as [39].

\[ P_{\text{pump}} = \frac{m \Delta p}{\rho_{\text{eff}}}, \]

where \( m \) is the mass flow rate and \( \Delta p \) is the pressure drop of the flow. By applying constant pumping power condition and employing no-slip and symmetrical boundary conditions at the wall and the center of the channel, respectively, Eq. (7) can be integrated to yield the Hagen-Poiseuille velocity profile as

\[ u = \frac{2\bar{u}_f}{C_\mu^{5/6}} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right], \]

where \( \bar{u}_f \) is the mean velocity of the base fluid over the cross-section area of the microchannel. For a constant pumping power, the mean velocity of the nanofluid, \( u \), can be related to \( \bar{u}_f \) as

\[ u = \frac{\bar{u}_f}{C_\mu^{5/6}}. \]

The nanofluid inside the microchannel is regarded as a continuum. To assess the validity of this assumption, we evaluate the Knudsen number \( Kn \) which is given by

\[ Kn = \frac{\lambda}{D}, \]

with \( \lambda \) the fluid molecule mean free path. The effective size and the mean free path of the water molecules are both of the same order of 0.3 \( \text{nm} \) [36]. In a similar manner, the mean free path of the nanoparticle is estimated to be the same order as its effective diameter of 60 nm. For the microchannel of interest with an inner diameter of 850 \( \mu \text{m} \), the Knudsen number is smaller than 7.1 \times 10^{-5}, justifying the continuum assumption.

By considering that the fluid phase and the nanoparticles are in thermal equilibrium with constant thermophysical properties, the energy equation is given by

\[
\rho_{\text{eff}} c_{p,\text{eff}} \frac{\partial T}{\partial x} = k_{\text{eff}} \frac{\partial^2 T}{\partial r^2} + k_{\text{eff}} \frac{\partial^2 T}{\partial x^2} + \mu_{\text{eff}} \left( \frac{\partial u}{\partial r} \right)^2, \quad (12)
\]

where \( T \) is the temperature of nanofluid. In Eq. (12), the streamwise conduction and the viscous dissipation terms are incorporated as the second term and the third term on the right-hand side, respectively. The radial thermal boundary conditions are expressed as

\[ k_{\text{eff}} \frac{\partial T}{\partial r} \bigg|_{r=r_0} = q_{\text{in}} = q_{\text{in}}^0 \exp(-\beta x), \quad \frac{\partial T}{\partial r} \bigg|_{r=0} = 0. \quad (13)\]

On account of Eq. (13), the microchannel is subjected to an exponentially decaying wall heat flux where \( q_{\text{in}}^0 \) is the initial wall heat flux at the position \( x = 0 \) and \( \beta \) is the exponent in wall heat flux variation. The exponential wall heat flux condition is more realistically practical compared to the idealized isothermal and isoflux boundary conditions which are the special cases of the exponential wall heat flux boundary condition [40]. The effective heat flux entering the working fluid from the solid wall is expressed in an exponential form [41,42]. Integrating Eq. (12) over the cross section of the microchannel gives

\[
\rho_{\text{eff}} c_{p,\text{eff}} \frac{\partial T}{\partial x} \int_{0}^{r_0} u Tr dr = k_{\text{eff}} \frac{\partial T}{\partial r} \bigg|_{r=0} + k_{\text{eff}} \frac{\partial^2 T}{\partial r^2} \int_{0}^{r_0} Tr dr + \mu_{\text{eff}} \int_{0}^{r_0} \left( \frac{\partial u}{\partial r} \right)^2 r dr. \quad (14)
\]

By defining the average temperature \( T \) as

\[ T = \frac{1}{A_r} \int_{A_r} T \, dA_r = \frac{1}{A_r} \int_{0}^{r_0} Tr \, dr, \quad (15)\]

and the bulk mean temperature as

\[ T_b = \frac{1}{A_r} \int_{A_r} uT \, dA_r = \frac{2}{A_r} \int_{0}^{r_0} uTr dr, \quad (16)\]

and applying the thermal boundary conditions in Eq. (13), with the assumption of \( T_b \approx T \), Eq. (14) can be expressed as [26].

\[
\rho_{\text{eff}} c_{p,\text{eff}} \frac{dT}{dx} = 2\pi r_0 q_{\text{in}} \exp(-\beta x) + k_{\text{eff}} r_0^3 \frac{dT}{dr^2} + 8\pi \mu_{\text{eff}} u^2. \quad (17)
\]
By introducing the following dimensionless variables
\[ X = \frac{x}{L}, \quad \xi = \frac{r_0}{L}, \quad \phi = \frac{\beta}{\mu}, \quad \theta = \frac{k_0(T - T_0)}{2r_0q_0}. \] (18)
with \( T_0 \) denoting the mean temperature at the entrance \((x = 0)\), Eq. (17) is nondimensionalized as
\[ C_a \frac{d\theta}{d\xi} = C_b \exp(-\beta X) + \frac{d^2\theta}{dX^2} + C_c, \] (19)
where
\[ C_a = \frac{\gamma Pe}{2C_{fs} \xi}, \quad C_b = \frac{1}{C_{fs} \xi^2}, \quad C_c = 8Br'C_b. \] (20)
In Eq. (20), the Peclet number is defined as
\[ Pe = \frac{2r_0 \mu L C_{p,f}}{k_f}. \] (21)
The modified Brinkman number is given by
\[ Br' = \frac{\mu L^2}{2C_f q_0}. \] (22)
and the heat capacity ratio is given by
\[ \gamma = \frac{\rho_{nf} C_{p,nf}}{\rho_{nf} C_{p,f}}. \] (23)
By specifying the temperature of nanofluid at the entrance and assuming the exit of the microchannel is connected to an adiabatic section, the non-dimensional axial thermal boundary conditions are given by
\[ \theta(0) = 0, \quad \frac{d\theta}{dX}(1) = 0. \] (24)

Solving Eq. (19) together with the thermal boundary conditions in Eq. (24) yields the closed-form dimensionless temperature profile as
\[ \theta_1(X) = \left\{ -C_a C_b \exp(-\beta X) - C_b \exp(C_b X - C_a - \beta) \right\}
+ C_a C_b \beta \exp(C_b X - \beta) \exp(C_b X - C_a) - C_b \beta \exp(-C_a) \exp(-C_b) \]
\[ -C_a C_b \beta \exp(C_b X - C_a) - \exp(-C_a) \right\} / \left(C_a^2 \beta + \beta \right). \] (25)
For the case when the streamwise conduction term is neglected in Eq. (19), i.e. \( d^2\theta/dX^2 = 0 \), the dimensionless temperature profile is obtained as
\[ \theta_2(X) = \frac{C_b \beta X - C_b \exp(-\beta X) - 1}{C_a \beta}. \] (26)

For the purpose of comparing the results, we denote the model with streamwise conduction term incorporated in the energy equation as Model 1 (with subscript 1) and the model without streamwise conduction term as Model 2 (with subscript 2).

To quantify the contribution of streamwise conduction and convection in the total heat transfer rate, Eq. (17) is integrated over an axial distance \( x' \) of the microchannel and it can be shown as
\[ \frac{\rho_{nf} c_{p,nf} \mu L}{C_{p,fs} \rho_{nf}} \int_0^{x'} \frac{dT_1}{dx} dx = 2\pi r_0 q_0 \int_0^{x'} \exp(-\beta x) dx \]
\[ + k_{eff} \frac{r_0^2}{\mu} \int_0^{x'} \frac{d^2T_1}{dx^2} dx + \frac{8\pi \mu_{eff} u_0^2}{C_{p,fs} \rho_{nf}} \int_0^{x'} dx. \] (27)

For an axial distance of \( x' \) from the entrance, the term on the left-hand side of Eq. (27) represents the cumulative convection heat transfer rate and the second term on the right-hand side is the cumulative streamwise conduction heat transfer rate. The first term on the right-hand side is the total heat input from the solid wall circumference for a distance of \( x' \) and the last term on the right-hand side is the cumulative viscous dissipative rate. In a more concise form, Eq. (27) can be rewritten as
\[ q_{con,1} = q_{in,1} + q_{cond,1} + q_{vol,1}. \] (28)
where
\[ q_{con,1} = \frac{\rho_{nf} c_{p,nf} \mu L}{C_{p,fs} \rho_{nf}} \left(T_{in} - T_0\right), \] (29)
\[ q_{in,1} = \frac{2\pi r_0 q_0}{\beta} \left(1 - \exp(-\beta x)\right), \] (30)
\[ q_{cond,1} = k_{eff} \frac{r_0^2}{\mu} \frac{dT_1}{dx} \bigg|_0^{x'}, \] (31)
\[ q_{vol,1} = \frac{8\pi \mu_{eff} u_0^2 x}{C_{p,fs} \rho_{nf}}, \] (32)
where \( T_{in} \) is the mean temperature of nanofluid at the axial distance \( x' \) for Model 1. From the energy balance as shown in Eq. (28), the heat input from the solid wall and the internal heat generation due to viscous dissipation are transported by means of convection and conduction in the axial direction. For Model 2 where \( q_{cond,1} \) is neglected, the equation in terms of heat transfer rate can be written as
\[ q_{con,2} = q_{in,2} + q_{vol,2}. \] (33)

The heat input is identical in both cases, i.e. \( q_{in,2} = q_{in,1} \). As depicted in Eq. (32), the viscous dissipative rate is hydrodynamically dependent on the viscosity and velocity of the nanofluid. This leads to \( q_{vol,2} = q_{vol,1} \). The only term in Model 2 different from those in Model 1 is the convection heat transfer rate, which is given by
\[ q_{con,2} = \frac{\rho_{nf} c_{p,nf} \mu L}{C_{p,fs} \rho_{nf}} \left(T_{in} - T_0\right), \] (34)
When the streamwise conduction is neglected, convection plays the exclusive role in transporting the heat generation from the solid wall and the viscous dissipation. Based on Eq. (27), the corresponding heat transfer rates for the entire length of the microchannel are given by
\[ q_{con,i} = \frac{\rho_{nf} c_{p,nf} \mu L}{C_{p,fs} \rho_{nf}} \left(T_{in} - T_0\right), \] (35)
\[ q_{in,i} = \frac{2\pi r_0 q_0}{\beta} \left(1 - \exp(-\beta L)\right), \] (36)
\[ q_{cond,i} = -k_{eff} \frac{r_0^2}{\mu} \frac{dT_1}{dx} \bigg|_0^L, \] (37)
\[ q_{vol,i} = \frac{8\pi \mu_{eff} u_0 L}{C_{p,fs} \rho_{nf}}, \] (38)

3. Results and discussion

The distinctive heat transfer characteristics between the microscale and the macro-scale channel flows arises from the
pronounced effects of the streamwise conduction and the viscous dissipation taking place in the former [27]. In micro-scale channel flows, the effect of viscous dissipation leads to drastic change in flow and temperature fields of fluids with low specific heats and high viscosities [26,43]. Employing water as working fluid in microchannels, the experimental data in laminar flow regime correlate well with the Brinkman number, which is defined as the ratio of the heat generation due to viscous forces to the heat transferred from the wall to the fluid [44]. When the variations of Reynolds number and the Prandtl number are marginal, small values of Brinkman number do not materially affect the heat transfer characteristics. The impact of viscous dissipation on the convective heat transfer characteristics is noticeable typically when the Reynolds number and the Prandtl number are high [43]. Subsequently, the Peclet number which is the product of the Reynolds number and the Prandtl number is also high. On the other hand, the effect of streamwise conduction is only prominent at low Peclet number when the Reynolds number and the Prandtl number are mutually low [19,21,24]. Since the present study is devoted to the effect of streamwise conduction of low-Peclet-number nanofluid flow in microchannels, the effect of viscous dissipation can be reasonably neglected.

All the results are reported for laminar flow of water-based nanofluids containing alumina nanoparticles and the input parameters required for the computation are listed in Table 1. The properties of water [39] and Al₂O₃ nanoparticles [45] are evaluated based on the mean temperature in the axial direction as

\[ T_{\text{op},i} = \frac{1}{L} \int_0^L T_i(x) \, dx, \quad i = 1, 2. \]  

(39)

As the temperature of nanofluid varies in the axial direction, iterations have been performed in order to obtain the \( T_{\text{op},i} \) of a specific operation condition for different values of Pe and \( \phi \) with a convergence criterion of \( 1 \times 10^{-6} \) K. The consideration of the axial conduction in the solid wall of the microchannel leads to a wall heat flux variation which can be expressed in exponential forms [41,42]. In this study, the exponential wall heat flux is characterized by an exponent \( \beta \) as depicted in Eq. (13).

3.1. Axial temperature distributions

For Pe = 10 and in the limit of \( \beta \to 0 \), \( \hat{T}_i(X) \) in Eq. (25) corresponds to the isoflux wall thermal condition. The dimensionless temperature profile for \( \phi = 0 \) is plotted in Fig. 1 and compared with the results reported in [26], by taking into account the streamwise conduction effect in the energy equation under the isoflux condition. An excellent agreement is observed between the two temperature profiles and the formulation of the present study is validated.

As constant pumping power is applied to all nanofluids regardless of the nanoparticle volume fractions, it is instructive to investigate the relationship of the pumping power and the Peclet number, which are the key parameters of determining the impact of streamwise conduction. Fig. 2 depicts the relationship between

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<td>Parameters</td>
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<td>Mean temperature at the entrance, ( T_0 )</td>
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<tr>
<td>Wall heat flux at the entrance, ( q_w^0 )</td>
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<tr>
<td>Dimensionless exponent in wall heat flux variation ( \hat{\beta} )</td>
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<td>Channel inner diameter</td>
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Fig. 1. Dimensionless temperature \( \hat{T} \) profiles for \( \hat{\beta} = 0 \), Pe = 10 and \( \phi = 0 \). Theoretical data from the model by Hetsroni et al. [26] is included for comparison.

Fig. 2. Variations of the pumping power with the Peclet number.

Fig. 3. Dimensionless temperature \( \hat{T} \) profiles with \( \phi \) as a parameter at (a) Pe = 5, and (b) Pe = 25.
the pumping power and the Peclet number. It is observed that the pumping power which increases in a parabolic manner with respect to the Peclet number, is a one-to-one function of the Peclet number. In this context, the Peclet number which is relevant in the study of streamwise conduction is used to characterize the flow in the following analysis.

To characterize the role of streamwise conduction in the nanofluid flow, a comparison between the models with and without considering streamwise conduction is performed by analyzing the deviations induced in the axial temperature distributions. Fig. 3a and b shows the dimensionless nanofluid temperature distributions in the axial direction for both models at \( \text{Pe} = 5 \) and \( \text{Pe} = 25 \), respectively, with the nanoparticle volume fraction as a parameter. The temperature of nanofluid increases in the vicinity of the entrance region with steep temperature gradient. Further downstream from the entrance, the temperature gradient decreases exponentially and the temperature approaches an asymptote, showing that the fluid flows isothermally at an asymptotic temperature without absorbing heat for the remaining of the channel. From Model 1, one can observe that the fluid temperature is lower than that of Model 2. In the presence of a temperature gradient, the conduction heat transport occurs in the direction of decreasing temperature which is opposing to the flow direction. Therefore, when the streamwise conduction term is neglected, the temperature of Model 2 is higher than that of Model 1. In addition, the discrepancy between the two models increases with \( \phi \), indicating that the heat transfer performance of nanofluid has been overestimated without considering the streamwise conduction, especially for the case of high \( \phi \). On the other hand, the base fluid (\( \phi = 0\% \)) shows relatively small discrepancy between the temperature profiles of the two models. Previous studies verified that the incorporation of streamwise conduction in the energy equation alters the thermal characteristics of low-Peclet-number flows [22,27,46]. In nanofluid flows, the effect of the streamwise conduction is further augmented. This augmentation is deemed to be attributable to the pronounced increase of effective thermal conductivity of nanofluid compared to that of its base fluid. The enhanced effective thermal conductivity induces significant effect on the streamwise conduction term \( C_l k (\partial^2 T / \partial x^2) \) and hence on the temperature distribution of the nanofluid. The fact that the effective thermal conductivity of nanofluid increases with \( \phi \) leads to more prominent deviation of the temperature profiles between Model 1 and Model 2. However, the deviation between the two models is greater when the Peclet number is lower as illustrated in Fig. 3a and b.

As depicted in Fig. 3, the temperature profile approaches asymptote after a distance from the vicinity of the entrance, indicating the existence of an adiabatic region where the fluid flows isothermally at an asymptotic temperature without absorbing heat. Heat transfer from the solid wall to the nanofluid takes place only in a distance preceding the adiabatic region, namely the effective heat transfer length \( L_{\text{eff}} \). In the present study, the effective heat transfer length is evaluated numerically using Newton–Raphson method by finding the axial distance from the entrance where the nanofluid temperature reaches 99.99% of the outlet temperature of the microchannel. This length is equivalent to the distance from the entrance where the temperature gradient first approaches zero. As shown in Eq. (31), the streamwise conduction is affected by the change of temperature gradient. The microchannel region beyond \( L_{\text{eff}} \) generates negligible streamwise conduction as the change in the temperature gradient beyond this length is close to zero. Here, we define the non-dimensional form of the effective heat transfer length as

\[
\tilde{L}_{\text{eff}} = \frac{L_{\text{eff}}}{L}.
\]
The effective thermal conductivity of nanofluid with higher φ. A water-Al₂O₃ nanofluid with φ = 10% imparts 206% higher of \( \psi_{\text{cond}} \) compared to that of its base fluid. Therefore, the effect of streamwise conduction is more significant in nanofluid flow at low Peclet number, especially in the vicinity of the entrance of the channel.

We proceed to parametrically study the variations of the total streamwise conduction as given in Eq. (37). A dimensionless term is defined as

\[
\psi_{\text{cond}} = \frac{Q_{\text{cond,1}}}{Q_{\text{in,1}}} \quad \text{(42)}
\]

which is the ratio of the total streamwise conduction heat transfer rate to the total heat input from the wall. Fig. 6a illustrates a surface plot of \( \psi_{\text{cond}} \) as a function of Peclet number and φ. A flatter surface is observed at low φ and high Peclet number region and on the other hand, the surface curves up at high φ and low-Peclet-number region. The surface is projected on the \( \psi_{\text{cond}} - P \) plane with φ as a parameter in Fig. 6b. It is obvious that the streamwise conduction plays a more significant role at low-Peclet-number region and its magnitude increases with φ. When Pe = 1, it is observed that 14.5% of the total heat input is transported by the streamwise conduction in the base fluid. For nanofluid of φ = 10%, the streamwise conduction contributes 32% of the heat transport. The contribution of streamwise conduction in the total heat transport is significantly amplified in nanofluid due to the fact that the effective thermal conductivity of nanofluid is intrinsically higher than that of its base fluid in light of the suspension of nanoparticles with higher thermal conductivity.

In addition to the effective thermal conductivity, the change of fluid temperature gradient also contributes to the streamwise conduction. As discussed earlier, the effective heat transfer length \( L_{\text{eff}} \) can be defined as the distance where the temperature gradient first approaches zero. This means that it can be used to define the length of the microchannel where the streamwise conduction is most prevalent. Fig. 7 depicts the \( \psi_{\text{cond}} \) as a function of \( L_{\text{eff}} \), with φ as the parameter. It is obvious that the streamwise conduction heat transfer rates increases with \( L_{\text{eff}} \). For a particular value of \( L_{\text{eff}} \), the nanofluid with higher φ induces higher \( \psi_{\text{cond}} \) due to the increase in its effective thermal conductivity.

To study the role of the convection heat transfer rate, we define a non-dimensional parameter as

\[
\psi_{\text{conv}} = \frac{Q_{\text{conv,1}}}{Q_{\text{in,1}}} \quad \text{(43)}
\]

which is the ratio of the total convection heat transfer rate to the total heat input. The dimensionless ratio \( \psi_{\text{conv}} \) is plotted as a function of φ and Peclet number in Fig. 8a and the corresponding \( \psi_{\text{cond}} - P \) projection is illustrated in Fig. 8b with φ as a parameter. We can observe that the convection heat transfer is more significant in the flow regime of higher Peclet number. In low-Peclet-number flow regime, the streamwise conduction plays a more discernible role.
role. As higher value of \( \phi \) induces greater \( \beta_{\text{cond}} \), the nanofluids with higher \( \phi \) show lower magnitude of \( \beta_{\text{conv}} \), as depicted in Fig. 8. However, when the Peclet number is increased, the effect of \( \phi \) become insensitive and \( \beta_{\text{conv}} \) approaches an asymptote indicating the heat transport is dominated by the convective heat transfer. Comparing to Model 2, it is evident that the heat transport is solely contributed by convective heat transfer when the streamwise conduction is neglected and the contribution of the convective heat transfer is over-estimated. The magnitude of the streamwise conduction increases with \( \phi \), showing that the degree of overestimation increases with \( \phi \). It can be concluded that the addition of nanoparticles in the base fluid enhances the streamwise conduction while on the other hand reduces the contribution of the convective heat transfer. Neglecting the streamwise conduction leads to erroneous heat transfer performance evaluation particularly at the low-Peclet-number flow regime.

We define a ratio \( M \) to investigate the interrelationship between the streamwise conduction and the convection heat transport rate as

\[
M = \frac{|Q_{\text{cond}}|}{|Q_{\text{conv}}|}.
\]

Fig. 9a illustrates a surface plot of the ratio \( M \) as a function of Peclet number and \( \phi \). The surface shows a relatively high curvature at low Peclet number. The surface is projected on the \( M - \text{Pe} \) plane with \( \phi \) as a parameter in Fig. 9b. It is obvious that the streamwise conduction is significant at low-Peclet-number region. For small Peclet number, \( M \) is significantly large, indicating that the streamwise conduction is comparable to the convective heat transfer. However, when Pe increases, \( M \) decreases drastically, showing that the convection heat transfer dominates and overcomes the streamwise conduction heat transfer. Beyond Pe = 10, \( M \) approaches an asymptote of 0.01 and the heat transported by the fluid is mainly due to convection. On the other hand, \( M \) increases with the nanoparticle volume fraction for all Peclet numbers. Therefore, the degree of significance of the effect of streamwise conduction decreases with Pe and increases with \( \phi \).

In order to investigate the effect of the dimensionless wall heat flux exponent \( \beta \) on the significance of streamwise conduction, the ratio \( M \) is plotted as a function of \( \beta \) for Pe = 5 in Fig. 10. Based on Eq. (13), the wall heat flux diminishes along the length of the microchannel and the magnitude of decaying increases with \( \beta \). When \( \beta = 0 \), the exponentially decaying wall heat flux reduces to a constant wall heat flux \( q_0 \). From Fig. 10, it is observed that when the value of \( \beta = 0 \), \( M \rightarrow 0 \), indicating that the streamwise conduction is marginally small for all value of \( \phi \) when the wall heat flux is a constant. This justifies the common practice of neglecting the streamwise conduction for internal flows with constant wall heat flux thermal boundary condition. Fig. 10 shows that \( M \) increases proportionally with \( \beta \) and the slope is higher when \( \phi \) increases. Therefore, the significance of streamwise conduction over convection heat transfer in the total heat transport rate increases with \( \beta \).

To compare the significance of the streamwise conduction of the nanofluid with respect to that of the base fluid, a parameter
which is the ratio of the nanofluid’s streamwise conduction heat transfer rate to the base fluid’s streamwise conduction heat transfer rate is defined. Fig. 11a illustrates the surface plot of \( \Omega \) as a function of Peclet number and \( \phi \). When \( \phi = 0 \), \( \Omega \) approaches unity and then increases with \( \phi \). This indicates that the increases in the magnitude of streamwise conduction is proportional to \( \phi \). Fig. 11b depicts the corresponding \( \Omega \) vs. Pe projection, with \( \phi \) as a parameter. The variation of \( \Omega \) is sensitive to the Peclet number at small Peclet number. The value of \( \Omega \approx 2.21 \) at Pe = 1 for \( \phi = 0.10 \) shows that a 10% volume fraction of \( \text{Al}_2\text{O}_3 \) nanoparticles amplifies the streamwise conduction up to 221%. A volume fraction of nanoparticle of 0.5% amplifies the streamwise conduction up to 110%. Due to the enhanced effective thermal conductivity of the nanofluid, it can be concluded that the effect of streamwise conduction on the nanofluid flow in microchannel heat sink is significant albeit not dominant particularly for small Peclet number and high nanoparticle volume fraction of the nanofluid.

4. Conclusions
Under constant pumping power condition, the nanofluid temperature distribution is a strong function of the Peclet number when the streamwise conduction is incorporated in the energy equation. The deviation of the nanofluid temperature distribution between the models with and without considering streamwise conduction effect increases as Peclet number decreases. The effect of streamwise conduction is significant in the vicinity of the microchannel inlet and increases with the nanoparticle volume fraction. The effective heat transfer length \( \ell_{\text{eff}} \) increases with the nanoparticle volume fraction when the streamwise conduction is prevalent. The heat transport by streamwise conduction decreases with the Peclet number while the reverse is true for the heat transfer by convection. Beyond Pe = 10, the heat transported by the fluid is mainly due to convection and the degree of significance of the effect of streamwise conduction decreases dramatically with Peclet number. The increase in volume fraction of nanoparticle amplifies the contribution of streamwise conduction compared to the base fluid. It can be concluded that the effect of streamwise conduction on the nanofluid flow in microchannel heat sink is significant albeit not dominant particularly for small Peclet number and high nanoparticle volume fraction of the nanofluid.

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References


